

Wormholes and Spacetime Foam. an approach to the cosmological constant and entropy

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Abstract

This paper summarizes the contribution presented at the IX Marcel Grossmann Meeting (Rome, July 2000). A simple model of spacetime foam, made by N Schwarzschild wormholes in a semiclassical approximation, is here proposed. The Casimir-like energy of the quantum fluctuation of such a model and its probability of being realized are computed. Implications on the Bekenstein-Hawking entropy and the cosmological constant are considered. A proposal for an alternative foamy model formed by N Schwarzschild-Anti-de Sitter wormholes is here considered.

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The term Spacetime Foam was used for the first time by J.A. Wheeler to indicate that spacetime may be subjected to quantum fluctuations in topology and metric at the Planck scale [1]. Since then a lot of work has been done, especially in the direction of string theory and of Euclidean Quantum Gravity, to see if a nontrivial vacuum can be considered for a quantum theory of gravitation. In this paper we will consider a different approach based on a proposed model of spacetime foam, made by N coherent Schwarzschild wormholes [2]. The main reason which has led to consider such a model is based on a Casimir energy computation described by the following Hamiltonian

$$\Delta E(M) = E(M) - E(0) = \frac{\langle \Psi | H_{\Sigma}^{Schw.} - H_{\Sigma}^{Flat} | \Psi \rangle}{\langle \Psi | \Psi \rangle} + \frac{\langle \Psi | H_{qt} | \Psi \rangle}{\langle \Psi | \Psi \rangle}. \quad (1)$$

The first term represents the difference of the Hamiltonians evaluated on the Schwarzschild and flat spacetime respectively and the second term has the same meaning of the first one, but it is evaluated on the boundary of the manifold. Ψ is a trial wave functional of the gaussian form restricted to the traceless-transverse sector (TT), which is gauge invariant. No matter contribution has been considered. The one-loop contribution shows that quantum fluctuations of the gravitational metric shift the minimum of the effective energy from flat space (the classical minimum for the energy) to a multi-wormhole configuration. Indeed the total energy contribution to one loop is

$$\Delta_{N_w} E(M) \sim -N_w^2 \frac{V}{64\pi^2} \frac{\Lambda^4}{e}, \quad (2)$$

where V is the volume of the system, Λ is the U.V. cut-off and N_w is the wormholes number [2–4]. This expression shows that a non-trivial vacuum of the multi-wormhole type is favoured with respect to flat space. It is important to remark that it is the N - coherent superposition of wormholes that it is privileged with respect to flat space and not the single wormhole, because the single wormhole energy contribution has an imaginary contribution in its spectrum: a clear sign of an instability. Nevertheless the presence of an unstable mode is necessary to have transition from one vacuum (the false one) to the other one (the true vacuum) [5]. Three consequences of this multiply connected spacetime are:

1. the event horizon area of a black hole is quantized and by means of the Bekenstein-Hawking relation [6,7], also the entropy of a black hole is quantized. In particular for a Schwarzschild black hole, one gets

$$M = \frac{\sqrt{N}}{2l_p} \sqrt{\frac{\ln 2}{\pi}}, \quad (3)$$

namely the black hole mass is quantized.

2. A cosmological constant is induced by vacuum fluctuations as shown by Eq. (2) whose value is

$$\Lambda_c = \frac{\Lambda^4 l_p^2}{N_w 8e\pi}. \quad (4)$$

When the area-entropy relation is applied to the de Sitter geometry, we obtain

$$\frac{3\pi}{l_p^2 \ln 2 N_w} = \Lambda_c. \quad (5)$$

3. Combining Eq. (4) and Eq. (5), one gets

$$\Lambda_c = \frac{\Lambda^4 l_p^2}{N_w 8e\pi} = \frac{3\pi}{l_p^2 \ln 2 N_w}. \quad (6)$$

This means that we have found a constraint on the U.V. cut-off

$$\Lambda^4 = \frac{24e\pi^2}{\ln 2 l_p^4}. \quad (7)$$

This last consequence, although very encouraging, because at first glance the model seems to be U.V. finite, needs a very careful examination and interpretation. In fact, since we have adopted a certain number of approximations, Eq. (7) could be only an artifact of the calculation scheme. It is interesting to note that Eq. (2) can be obtained at least for $N_w = 1$ even for Schwarzschild-Anti-de Sitter wormholes [8]. In this case, the Hamiltonian is

$$\Delta E(M, b) = E(M, b) - E(b) = \frac{\langle \Psi | H_{\Sigma}^{S-AdS} - H_{\Sigma}^{AdS} | \Psi \rangle}{\langle \Psi | \Psi \rangle} + \frac{\langle \Psi | H_{ql} | \Psi \rangle}{\langle \Psi | \Psi \rangle}, \quad (8)$$

where $b = \sqrt{-3/\Lambda_c}$, the first term represents the difference of the Hamiltonians evaluated on the Schwarzschild-Anti-de Sitter and Anti-de Sitter spacetime respectively and the second term always represents a subtraction procedure on the boundary of Σ . This means that a selection rule has to emerge to compare the quantity

$$\Gamma_{\text{N-S-AdS holes}} = \frac{P_{\text{N-S-AdS holes}}}{P_{\text{AdS}}} \simeq \frac{P_{\text{foam}}}{P_{\text{AdS}}} \quad (9)$$

with

$$\Gamma_{\text{N-holes}} = \frac{P_{\text{N-holes}}}{P_{\text{flat}}} \simeq \frac{P_{\text{foam}}}{P_{\text{flat}}}. \quad (10)$$

In both cases, we find a non-vanishing probability that a non-trivial vacuum has to be considered. However in the case of Eq. (10), we have obtained that [2]

$$\Gamma_{\text{N-holes}} = P \sim \left| \exp \left(\frac{\Lambda_c}{8\pi l_p^2} V_c \right) (\Delta t) \right|^2, \quad (11)$$

which for the de Sitter case becomes

$$\exp \left(3\pi/l_p^2 \Lambda_c \right). \quad (12)$$

Thus we recover the Hawking result about the cosmological constant approaching zero [9]. Note that the vanishing of Λ_c is related to the growing of the wormholes number. Although these results are quite encouraging, an important question comes into play: why a space-time formed by wormholes has to be preferred with respect to the flat one, when the last one is the one we observe. The answer that at this stage can only be conjectured is that if we consider the following expectation value on the foam state

$$\frac{\langle \Psi_F | \hat{g}_{ij} | \Psi_F \rangle}{\langle \Psi_F | \Psi_F \rangle}, \quad (13)$$

when the number of wormholes is large enough, i.e. the scale is sufficiently large, we should have to obtain

$$\frac{\langle \Psi_F | \hat{g}_{ij} | \Psi_F \rangle}{\langle \Psi_F | \Psi_F \rangle} \rightarrow \eta_{ij}, \quad (14)$$

where η_{ij} is the flat space metric. This is a test that this foamy model has to pass if phenomenological aspects have to be considered.

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